

MARK SCHEME for the May/June 2008 question paper

9709/01

9709 MATHEMATICS

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

- AEF Any Equivalent Form (of answer is equally acceptable)
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- CWO Correct Working Only often written by a 'fortuitous' answer
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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<p>1 Use of sine rule $\frac{12}{\sin 45} = \frac{x}{\sin 60}$</p> <p>$\sin 60 = \frac{\sqrt{3}}{2}$ and $\sin 45 = \frac{1}{\sqrt{2}}$</p> <p>$\rightarrow BC = 6\sqrt{3} \cdot \sqrt{2}$ or $6\sqrt{6}$ or $\sqrt{216}$.</p>	<p>M1</p> <p>B1</p> <p>A1</p> <p>[3]</p>	<p>Used correctly in their triangle ABC</p> <p>Both of these correct</p> <p>Co – must be in surd form.</p>
<p>2 (i) $2 \tan^2 \theta \cos \theta = 3$</p> <p>Replaces $\tan^2 \theta$ by $\frac{\sin^2 \theta}{\cos^2 \theta}$ and</p> <p>Replaces $\sin^2 \theta$ by $1 - \cos^2 \theta$</p> <p>$\rightarrow 2\cos^2 \theta + 3\cos \theta - 2 = 0$</p> <p>(ii) Soln of quadratic $\rightarrow \frac{1}{2}$ and -2</p> <p>$\rightarrow 60^\circ$ and 300°</p>	<p>M1</p> <p>M1</p> <p>[2]</p> <p>M1</p> <p>A1 A1√</p> <p>[3]</p>	<p>For correct formula used</p> <p>For correct formula used</p> <p>Correct method of solving quadratic</p> <p>A1 for 60, A1√ for $(360 - 1st\ answer)$ and no other solutions in the range.</p>
<p>3 (i) $(2 + x^2)^5 = 2^5 + 5 \cdot 2^4 \cdot x^2 + 10 \cdot 2^3 \cdot x^4$</p> <p>$\rightarrow 32 + 80x^2 + 80x^4$</p> <p>(allow 2^5 for 32)</p> <p>(ii) $(1 + x^2)^2 = 1 + 2x^2 + x^4$</p> <p>Product has 3 terms in x^4</p> <p>$\rightarrow 80 + 160 + 32 = 272$</p>	<p>$3 \times B1$</p> <p>[3]</p> <p>B1</p> <p>M1</p> <p>A1√</p> <p>[3]</p>	<p>If coeffs ok but x and x^2, allow B1 special case. Allow 80, 80 if in (ii).</p> <p>Anywhere.</p> <p>Must be attempt at more than 1 term.</p> <p>For follow-through on both expansions, providing there are 3 terms added.</p>
<p>4 (i) Eliminates y to get</p> <p>$2x^2 - 7x + 6 = 0$ or $2y^2 - 5y + 3 = 0$</p> <p>$\rightarrow (2x - 3)(x - 2) = 0$</p> <p>$\rightarrow x = 2$ or $1\frac{1}{2}$</p> <p>(ii) $dy/dx = 4x - 8$</p> <p>$= 0$</p> <p>$x = 2$</p> <p>or completes the square and states stationary at $x = 2$.</p>	<p>M1 A1</p> <p>DM1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>[3]</p>	<p>y (or x) must be eliminated completely</p> <p>Setting 3 term quadratic to 0 + soln</p> <p>Both correct.</p> <p>Attempt to differentiate</p> <p>Setting differential to 0.</p> <p>co</p>

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<p>5 (i) Pythagoras $\rightarrow OT = 13, QT = 8$ cm Angle $POQ = \tan^{-1}(12/5) = 1.176$ $S = r\theta \rightarrow 5.88$ \rightarrow Perimeter = 25.9 cm</p> <p>(ii) Area of sector = $\frac{1}{2}r^2\theta$ used Area of triangle $OPT = \frac{1}{2} \times 12 \times 5$ Shaded area = $30 - 12.5 \times 1.176$ $\rightarrow 15.3 \text{ cm}^2$</p>	<p>B1 M1 M1 A1 [4]</p> <p>M1 B1 A1 [3]</p>	<p>For QT in either part. Could use \sin^{-1} or \cos^{-1} – in (i) or (ii) For $5 \times$ angle in rads or equivalent in $^\circ$ co</p> <p>Correct formula used. Anywhere co</p>
<p>6 $x \mapsto (3x + 2)^3 - 5$</p> <p>(i) $f'(x) = 9(3x + 2)^2$ or $81x^2 + 108x + 36$. Because of ()² always +ve Therefore an increasing function.</p> <p>(ii) $y = (3x + 2)^3 - 5$ $\sqrt[3]{y + 5} = 3x + 2$ $f^{-1}(x) = \frac{\sqrt[3]{x + 5} - 2}{3}$ Domain of $f^{-1} =$ range of f $\rightarrow x \geq 3$</p>	<p>B2,1,0 B1√ [3]</p> <p>M1</p> <p>A2,1</p> <p>B1 [4]</p>	<p>for $3(2x + 2)^2, \times 3$ and $d/dx(-5) = 0$ Allow for $k(3x + 2)^2$. Tries numbers B0.</p> <p>Attempt at making x the subject and completing to $y = \dots$</p> <p>Loses one mark for each error. Leaving answer as $f(y)$ is 1 error. co</p>
<p>7 (i) $a = 81, ar^3 = 24$ $\rightarrow r^3 = 24/81 \rightarrow r = \frac{2}{3}$ or 0.667</p> <p>(ii) $S_\infty = \frac{a}{1-r} = 81 \div \frac{1}{3} = 243$</p> <p>(iii) 2nd term of GP = $ar = 81 \times \frac{2}{3} = 54$ 3rd term of GP = $ar^2 = 36$ $\rightarrow 3d = -18$ ($d = -6$) $\rightarrow S_{10} = 5 \times (108 - 54) = 270$</p>	<p>M1 A1 [2]</p> <p>M1 A1√ [2]</p> <p>M1 M1 A1 [3]</p>	<p>Valid method for r. co</p> <p>Correct formula. √ for his a and r, providing $-1 < r < 1$.</p> <p>Finding the 2nd and 3rd terms of GP. M for finding d + correct S_{10} formula. co</p>

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<p>8 $f: x \mapsto 4x - 2k, g: x \mapsto \frac{9}{2-x}$</p> <p>(i) $fg(x) = \frac{36}{2-x} - 2k = x$</p> $x^2 + 2kx - 2x + 36 - 4k$ $(2k - 2)^2 = 4(36 - 4k)$ $k = 5 \text{ or } -7$ <p>(ii) $x^2 + 8x + 16 = 0, x^2 - 16x + 64 = 0$</p> $x = -4 \text{ or } x = 8.$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>A1 A1</p> <p>[3]</p>	<p>Knowing to put g into f (not gf)</p> <p>Correct quadratic.</p> <p>Any use of $b^2 - 4ac$ on quadratic = 0</p> <p>Both correct.</p> <p>Substituting one of the values of k.</p>
<p>9 $\frac{dy}{dx} = \frac{k}{x^3}$</p> <p>(i) Integrating $y = -k \frac{x^{-2}}{-2} (+c)$</p> <p>Sub (1,18) $18 = \frac{k}{2} + c$</p> <p>Sub (4,3) $3 = \frac{k}{32} + c$</p> $\rightarrow k = 32, c = 2$ <p>(ii) Area = $\left[-\frac{16}{x} + 2x \right]$ from 1 to 1.6</p> $\rightarrow [-10 + 3.2] - [-16 + 2] = 7.2$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>B1 B1</p> <p>M1 A1</p> <p>[4]</p>	<p>ok unsimplified</p> <p>Substitutes once (even if without + c)</p> <p>2nd substitution and solution of simultaneous equations for k and c</p> <p>co</p> <p>co</p> <p>Use of limits in an integral. co.</p>
<p>10 $\mathbf{OA} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \mathbf{OB} = 3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}$</p> <p>(i) $(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (3\mathbf{i} - 2\mathbf{j} + p\mathbf{k}) = 0$</p> $\rightarrow 6 - 2 + 2p = 0$ $\rightarrow p = -2$ <p>(ii) $(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k})$</p> $\rightarrow 6 - 2 + 12 \text{ allow for } \pm \text{ this}$ $= \sqrt{9} \times \sqrt{49} \cos \theta$ $\rightarrow \theta = 40^\circ$ <p>(iii) $\mathbf{AB} = \mathbf{i} - 3\mathbf{j} + (p - 2)\mathbf{k}$</p> $1^2 + 3^2 + (p - 2)^2 = 3.5^2$ $\rightarrow p = 0.5 \text{ or } 3.5$	<p>M1</p> <p>A1</p> <p>[2]</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>[4]</p>	<p>For $x_1x_2 + y_1y_2 + z_1z_2$ (in (i) or (ii))</p> <p>co</p> <p>nb Part (ii) gains 4 marks if (i) missing. co (M1 here if (i) not done)</p> <p>All connected correctly</p> <p>co</p> <p>Must be for AB, not BA.</p> <p>Pythagoras (allow if $\sqrt{\quad}$ wrong once)</p> <p>Method of solution.</p> <p>co</p> <p>(use of BA can score the last 3 marks)</p>

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<p>11 (i) Gradient of $AC = -\frac{1}{2}$ Perpendicular gradient = 2 Eqn of BX is $y - 2 = 2(x - 2)$ Sim Eqns $2y + x = 16$ with $y = 2x - 2$ $\rightarrow (4, 6)$</p> <p>(ii) X is mid-point of BD, D is $(6, 10)$</p> <p>(iii) $AB = \sqrt{(14^2 + 2^2)} = \sqrt{200}$ $BC = \sqrt{(2^2 + 6^2)} = \sqrt{40}$ \rightarrow Perimeter = $2\sqrt{200} + 2\sqrt{40}$ \rightarrow Perimeter = 40.9</p>	<p>B1 M1 M1 A1 [4]</p> <p>M1 A1√ [2]</p> <p>M1 DM1 A1 [3]</p>	<p>Correct gradient. Use of $m_1 m_2 = -1$ Correct form of equation co</p> <p>Any valid method. ft on (i).</p> <p>Use of Pythagoras once. 4 lengths added. co</p>
<p>DM1 for quadratic. Quadratic must be set to 0. Factors. Attempt at two brackets. Each bracket set to 0 and solved. Formula. Correct formula. Correct use, but allow for numerical slips in b^2 and $-4ac$.</p>		